$\sum_{n=1}^{\infty} \frac{1}{n^n} = 1 + \frac{1}{4} + \frac{1}{27} + \frac{1}{256} + \cdots$ $\lim_{n\to\infty} \left| \frac{(1)^{1/n}}{(n^n)^{1/n}} \right| = \lim_{n\to\infty} \frac{1}{n} = 0 < 1$ Converges absolutely

What you'll Learn About
Root Test

$r = -\frac{1}{10}$
$|r| < 1$
Converges absolutely

A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{10^n} = -\frac{1}{10} + \frac{1}{100} - \frac{1}{1000} + \frac{1}{10000}$

B) $\boxed{\sum_{n=1}^{\infty} \frac{(1)}{n^n}}$

$\cancel{\lim_{n\to\infty} \sqrt[n]{\frac{(-1)^n}{10^n}}} =$

$\lim_{n\to\infty} \left| \sqrt[n]{\frac{(-1)^n}{10^n}} \right| = \lim_{n\to\infty} \left| \left(\frac{(-1)^n}{10^n} \right)^{1/n} \right| = \lim_{n\to\infty} \left| \frac{((-1)^n)^{1/n}}{(10^n)^{1/n}} \right| = \frac{1}{10}$

$\frac{1}{10} < 1$ Converges absolutely

C) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+10} \right)^n$

$\lim_{n\to\infty} \left| \left(\left(\frac{n}{3n+10} \right)^n \right)^{1/n} \right| = \lim_{n\to\infty} \frac{n}{3n+10} = \frac{1}{3} < 1$

Converges Absolutely

D) $\sum_{n=1}^{\infty} \left(\frac{5n}{3n+10} \right)^{2n}$

$\lim_{n\to\infty} \left| \left(\left(\frac{5n}{3n+10} \right)^{2n} \right)^{\frac{1}{n}} \right| = \left(\frac{5}{3} \right)^2$

$\left(\frac{5}{3} \right)^2 > 1$

Diverges

E) $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+10} \right)^n$

$\lim_{n\to\infty} \left(\left| \frac{-2n}{n+10} \right|^n \right)^{1/n} = 2 > 1$

Diverges

What you'll Learn About
Alternating Series Test

$A)\ \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

$n = 100$ $S_{100} = 1.6349839$

$n = 1000$ $S_{1000} = 1.643934567$

$n = 10000$ $S_{10000} = 1.644834072$

converge

$p = 2 > 1$

$B)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots$

$n = 100$ $S_{100} = -.8224175334$

$n = 1000$ $S_{1000} = -.8224665339$

$n = 10000$ $S_{10000} = -.822467028$

converges Absolutely

$C)\ \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

Harmonic Diverges

$S_{100} = 5.187377518$

$S_{1000} = 7.485470861$

$S_{10000} = 9.787606036$

$D)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

conditionally converges

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

Alternating Harmonic Converges

$S_{100} = -.6881721793$

$S_{1000} = -.6926474306$

$S_{10000} = -.6930971831$

Conditional Convergence Series alternates and the terms decrease in absolute value to zero. Conditionally Converges	E) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}} =$ i) $\lim_{n\to\infty} \left\| \frac{(-1)^n}{\sqrt{n^2+1}} \right\| = 0$ ii) $\left\| \frac{(-1)^{n+1}}{\sqrt{(n+1)^2+1}} \right\| < \left\| \frac{(-1)^n}{\sqrt{n^2+1}} \right\|$ F) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}} =$ i) $\lim_{n\to\infty} \left\| \frac{(-1)^{n-1}}{n^{1/3}} \right\| = 0$ ii) next < previous $\left\| \frac{(-1)^{(n+1)-1}}{(n+1)^{1/3}} \right\| < \left\| \frac{(-1)^{n-1}}{n^{1/3}} \right\|$ G) $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1} =$ i) $\lim_{n\to\infty} \left\| \frac{(-1)^n n^4}{n^3+1} \right\| \neq 0$ Diverges

Absolute? NO

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ Diverge

Compare to Harmonic series which diverges

$\lim_{n\to\infty} \frac{\left(\frac{1}{\sqrt{n^2+1}}\right)}{\left(\frac{1}{n}\right)} = 1$

$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$

Absolute?

$\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

$p = \frac{1}{3} \leq 1$

Diverges

$\lim_{n\to\infty} \frac{(-1)^n n^4}{n^3+1} \neq 0$